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SECULAR EVOLUTION OF HD 12661: A SYSTEM CAUGHT AT AN UNLIKELY TIME

Dimitri Veras[1], Eric B. Ford[1]

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ABSTRACT

The eccentricity evolution of multiple planet systems can provide valuable constraints on planet formation models. Unfortunately, the inevitable uncertainties in the current orbital elements can lead to significant ambiguities in the nature of the secular evolution. Integrating any single set of orbital elements inadequately describes the full range of secular evolutions consistent with current observations. Thus, we combine radial velocity observations of HD 12661 with Markov Chain Monte Carlo sampling to generate ensembles of initial conditions for direct n-body integrations. We find that any mean motion resonances are quite weak and do not significantly impact the secular evolution, and that current observations indicate circulation or large amplitude libration of the periapses. The eccentricity of the outer planet undergoes large oscillations for nearly all of the allowed two-planet orbital solutions. This type of secular evolution would arise if planet c had been impulsively perturbed, perhaps due to strong scattering of an additional planet that was subsequently accreted onto the star. Finally, we note that the secular evolution implied by the current orbital configuration implies that planet c spends $\sim$ 96% of the time following an orbit more eccentric than that presently observed. Either this system is being observed during a relatively rare state, or additional planets are affecting the observed radial velocities and/or the system's secular eccentricity evolution.

Subject headings: celestial mechanics — stars: individual (HD 12661) — planetary systems: formation — methods: n-body simulations, statistical

1. INTRODUCTION

The secular evolution of multi-planet systems has become a topic of considerable importance for constraining planetary formation theories (Ford et al. 2005; Adams & Laughlin 2006; Barnes & Greenberg 2006b; Sándor et al. 2007). We investigate the secular evolution of the two giant planets orbiting HD 12661, a $\simeq 1.136 M_{\odot}$ star. Planet HD 12661 b has a semimajor axis of $\simeq$ 0.83AU and a moderate eccentricity, and planet HD 12661 c follows a nearly circular orbit about three times further away (Fischer et al. 2003). This system has inspired several dynamical studies. A brief survey of their results demonstrates the importance of considering the uncertainty in the current orbital configuration. Kiseleva-Eggleton et al. (2002) considered coplanar edge-on systems and found that the then-current best-fit solution was chaotic, behavior that is likely due to close proximity to the 9:2 mean-motion resonance (MMR). Both Goździewski (2003) and Lee & Peale (2003) found that the system was close to the 11:2 MMR and that the periapses underwent large amplitude libration for the edge-on case, as well as for a broad range of inclinations. Zhou & Sun (2003) claimed that aligned configurations were less likely to be chaotic and thus more likely to be stable. Goździewski (2003) found the system could be chaotic, but still stable for $\sim$Gyr thanks to a secular apsidal lock about an antialigned configuration. Goździewski & Maciejewski (2003) performed an independent analysis of the Fischer et al. (2003) radial velocities to find a better orbital fit that placed the system very near (and likely in) the 6:1 MMR that could be stabilized by secular apsidal lock for a broad range of inclinations.

Electronic address: veras@astro.ufl.edu
[1] Astronomy Department, University of Florida, 211 Bryant Space Sciences Center, Gainesville, FL 32111, USA

Despite the different orbital solutions, both Lee & Peale (2003) and Goździewski & Maciejewski (2003) found that the periapses were more likely to librate about an antialigned configuration, but that librations about an aligned configuration were also possible, particularly for systems with large inclinations. However, Ji et al. (2003) reported that aligned and antialigned configurations were nearly equally likely. Subsequently, Butler et al. (2006) published an orbital solution with a ratio of orbital periods approaching 13:2. Barnes & Greenberg (2006a) found that this orbital solution places the system very near the the boundary between librating and circulating modes of secular evolution. The revised orbital solutions for HD 12661 b & c and the lack of consensus regarding the system's secular evolution both motivate an updated dynamical study. Further, the historical range of orbital configurations illustrates the importance of properly accounting for uncertainties in orbital determinations.

Goździewski (2003) found that the classical secular theory gave only a crude approximation to the secular evolution due to large eccentricities and the 11:2 MMR. Both Veras & Armitage (2007) and Libert & Henrard (2007) caution against using low-order secular theories to model the system's behavior. Lee & Peale (2003), Rodríguez & Gallardo (2005), and Libert & Henrard (2007) found that either the octupole or high-order Laplace-Lagrange approximation described the secular evolution of HD 12661 well and that the secular dynamics is not significantly affected by the 5:1, 11:2, and 6:1 MMRs. Although we do not expect near-resonant terms to significantly affect the secular evolution, we use direct n-body integrations to account for the full dynamics, including all possible MMRs.

We determine the mode(s) of secular evolution consistent with observations and the requirement of long-term stability. Our study improves upon previous studies by

combining a Bayesian analysis of an updated set of radial velocity observations with direct n-body integrations to examine HD 12661's stability and secular evolution. We characterize the full range of orbital histories that are consistent with observations and reject solutions that do not exhibit long-term orbital stability. Thus, we can make quantitative statements about the relative probability of different modes of secular evolution.

2. METHODS

We reanalyze an updated set of radial velocity observations (Wright et al. 2008) assuming that the stellar velocity is the superposition of two Keplerian orbit plus noise. First, we perform a brute force search over parameter space to verify that there are no qualitatively different and comparably likely solutions. Working in a Bayesian framework, we generate an ensemble of $\simeq 5 \times 10^5$ orbital solutions using Markov Chain Monte Carlo (MCMC; Ford 2005, 2006; Gregory 2007a,b). Each state of the Markov chain includes the orbital period (P), velocity amplitude (K), eccentricity (e), argument of pericenter measured from the plane of the sky (ω), and mean anomaly at a given epoch (u) for each planet. We use the priors, candidate transition functions, automated step size control, and convergence tests described in Ford (2006). In particular, we assume a prior for the velocity "jitter" (σ_j) of $p(\sigma_j) \propto [1 + \sigma_j/(1\mathrm{m}s^{-1})]^{-1}$.

We randomly select subsamples of orbital solutions to be investigated with long-term n-body integrations and consider a range of possible inclinations (i) and ascending nodes (Ω). We generate each orbit from an isotropic distribution (i.e., uniform in $\cos i$ and Ω) and divide the resulting systems into bins based on the relative inclination between the two orbital planes ($i_{\rm rel}$). The bins are 1) $i_{\rm rel} = 0°$ (coplanar), 2) $0° \leq i_{\rm rel} \leq 30°$, 3) $30° \leq i_{\rm rel} \leq 60°$, 4) $60° \leq i_{\rm rel} \leq 90°$, 5) $90° \leq i_{\rm rel} \leq 120°$, 6) $120° \leq i_{\rm rel} \leq 150°$, and 7) $150° \leq i_{\rm rel} \leq 180°$. We integrate 2,000 systems in each of the first four bins and 500 in each of the last three bins. We can reweight our simulations so as to determine the probability of the different modes of secular evolution for various assumed distributions of relative inclinations. From each set $(P, K, e, \omega, i, \Omega, u)$, we generate the planet mass (m) and semi-major axis (a) using a Jacobi coordinate system (Lee & Peale 2003).

We used the hybrid symplectic integrator of `Mercury` (Chambers 1999) to integrate each set of initial conditions for at least 1 Myr. Based on a smaller series of 10 Myr integrations, we found that the vast majority of instabilities are manifest within 1 Myr. We classified systems as "unstable" if, for either planet, $a_{\max} - a_{\min} > \tau a_i$, where $a_{\max}$ and $a_{\min}$ represent the maximum and minimum values of the semimajor axis, a_i is the initial value of the semimajor axis, and $\tau = 0.3$. We discarded each set of initial conditions that was found to be unstable, and analyzed the properties of the remaining "stable" systems. Although a small fraction of our "stable" systems might exhibit instability if integrated for much longer timescales, our criteria avoids miscategorizing a system exhibiting bounded chaos (e.g., Goździewski 2003) as unstable. We manually verified that the above criteria gives reasonable results and that various choices of $\tau \in [0.10, 0.30]$ make just a few percent difference in the number of simulations labeled as stable. The percent of stable systems in our 7 bins are: 1) 100%, 2) 99.7%, 3) 93.3%, 4) 16.4%, 5) 0.2%, 6) 16.7%, and 7) 98.6%. We independently affirmed the trend exhibited by these stability percentages by performing a smaller, additional set of simulations by using an ensemble of initial conditions generated from a different MCMC code that accounted for planet-planet interactions with a Hermite integrator.

3. RESULTS

Several previous studies of HD 12661 (e.g. Ji et al. 2003; Lee & Peale 2003; Zhou & Sun 2003) and other planetary systems (Chiang et al. 2001; Malhotra 2002; Ford et al. 2005; Barnes & Greenberg 2006b) have highlighted the importance of the apsidal angle ($\Delta\varpi = \Omega_b + \omega_b - \Omega_c - \omega_c$), which is useful for describing the secular dynamics of systems with small relative inclinations. Since we consider systems with a wide range of relative inclinations, we instead focus on $\Delta\varpi'$, the angle between the periastron directions projected onto the invariable plane. Typically, the apsidal angle is classified as circulating, librating about 0°, or librating about 180°. By inspecting plots of apsidal angle evolution for individual systems, we find that some systems spend most of the time librating about one center, but occasionally the apsidal angle circulates for a short period of time. Therefore, we consider two summary statistics describing the secular evolution: the root mean square (RMS) of $\Delta\varpi'$ and the mean absolute deviation (MAD) of $\Delta\varpi'$ about the libration center, where the "libration center" is defined as the angle about which the RMS $\Delta\varpi'$ is minimized. For a system undergoing small amplitude libration about either center, the MAD and RMS deviation are of order the libration amplitude. For a system undergoing uniform circulation, the MAD approaches 90° and the RMS deviation approaches 103.92°. These statistics can be used to identify the mode of secular evolution in clear cut cases and provide a quantitative measure that is well-defined even for systems with complex evolution.

We calculate both measures using orbital elements measured every 1000 yr. Although the libration amplitude (maximum absolute deviation of $\Delta\varpi'$) can be sensitive to the frequency of sampling and leads to ambiguities, both RMS $\Delta\varpi'$ and MAD $\Delta\varpi'$ are more robust statistics that allows us to describe the secular evolution with a simple quantitative measure, regardless of whether the system is librating, circulating, or switching between regimes due to short-term perturbations. This robustness is particularly important for studying systems where there is little distinction between the librating and circulating regimes, due to one planet's orbit periodically becoming nearly circular.

For each n-body integration, we determine the libration center and calculate both the RMS and MAD of $\Delta\varpi'$ about that center. We discover that the RMS $\Delta\varpi'$ ranges from $68° - 85°$ and MAD $\Delta\varpi'$ ranges from $63° - 82°$, depending on the relative inclination (see Table 1). The libration centers are preferentially anti-aligned for prograde, nearly-coplanar systems, and transition to almost entirely aligned for near-coplanar retrograde systems. The stability is highly dependent on initial relative inclination; very few highly inclined retrograde systems were stable, indicating that non-secular perturbations influence the long-term dynamics for some relative inclinations. Although large values of RMS $\Delta\varpi'$ and MAD $\Delta\varpi'$

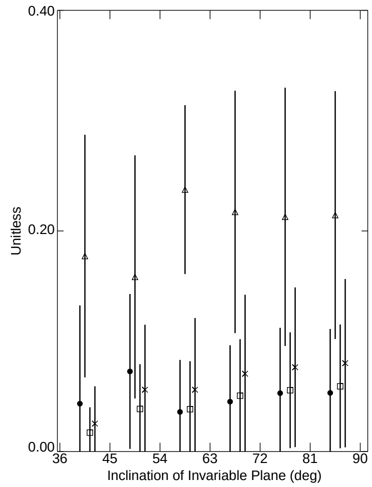

FIG. 1.— Values of c_1 (dots), c_2 (triangles), c_3 (squares), and c_4 (crosses) and their standard deviations, for coplanar systems, binned according to the sine of the inclination of the invariable plane.

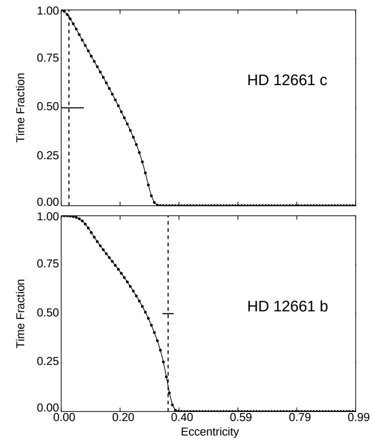

FIG. 2.— Fraction of time for which each planet's eccentricity exceeded a given threshold (x-axis) averaged over allowed orbital solutions. The curves with dots show results for the coplanar case, but assuming an isotropic distribution of inclinations relative to the plane of the sky. The dashed vertical lines indicate the median eccentricity values (0.031 and 0.361) for the planets' initial eccentricities, and their 5%-95% uncertainty ranges are indicated by the horizontal solid segments. HD 12661 c's orbital eccentricity exceeds its current value for approximately 96% of its evolution.

may be consistent with non-uniform circulation, visual spot checks of individual systems all suggest libration.

If a system of two planets on nearly circular orbits is excited impulsively, then the secular evolution will follow the boundary between the circulating and librating regimes and cause one orbit to repeatedly return to a nearly circular orbit (Malhotra 2002). In some systems, this secular evolution can be used to constrain the system's formation (Ford et al. 2005). Barnes & Greenberg (2006b) found that the Butler et al. (2006) orbital solution implies the eccentricity of HD 12661 c comes particularly close to zero, as parameterized by their ϵ parameter (Eq. 1 of Barnes & Greenberg 2006b). Because they considered only the published orbital solution in an edge-on, coplanar orientation, they were unable to asses the finding's robustness to uncertainties in the orbital parameters or non-edge-on systems. In order to determine what fraction of stable systems consistent with observations result in one planet returning to a nearly circular orbit, we calculate four statistics from each of our n-body integrations. The first two are $c_1 \equiv e_{min,c}/e_{max,c}$ and $c_2 \equiv e_{min,b}/e_{max,b}$, which represent the ratio of the minimum to maximum eccentricity for each planet. The third (c_3) is equal to $[2\min(e_b e_c)]/(x_{max}-x_{min}+y_{max}-y_{min})$, as defined by Barnes & Greenberg (2006b), where $x \equiv e_b e_c \sin(\Delta\varpi)$ and $y \equiv e_b e_c \cos(\Delta\varpi)$. The fourth is $c_4 \equiv \min(e_b e_c)/\max(e_b e_c)$, which is a similar measure that includes the eccentricities of both planets in a rotationally symmetric manner that is independent of the systems' orientation (unlike c_3).

Figure 1 plots the mean and standard deviation of each measure (c_1 - dots, c_2 - triangles, c_3 - squares, c_4 - crosses) for coplanar systems which have been binned according to the sine of the inclination of the invariable plane. The values c_1, c_3 and c_4 all maintain values < 0.1 for all bins of relative inclination for prograde systems, and < 0.15 for retrograde systems. The large values of c_2 (triangles) imply that HD 12661 b maintains a moderately eccentric orbit throughout the planet's evolution. Regardless, the measure c_4 demonstrates that the system lies near the boundary of libration and circulation. For the vast majority of systems, c_3 is over one order of magnitude greater than 0.003 (as found by Barnes & Greenberg 2006b).

The finding that planet c undergoes large eccentricity oscillations is remarkable since it currently has an eccentricity of only $\simeq$ 0.02. We investigate this observation further by plotting the fraction of simulation time that each planet's eccentricity exceeded a given value for the coplanar systems (Fig. 2). The dotted vertical lines indicate representative values for the planets' initial eccentricities. HD 12661 c's orbital eccentricity exceeds its current value for approximately 96% of its evolution. Thus, the near-circular orbit we now observe is a rare occurrence.

4. CONCLUSION

Several previous studies had suggested HD 12661 b & c are in a secular apsidal lock and are undergoing large amplitude librations. However, previously, even the orbital period of HD 12661 c was poorly determined. Now that radial velocity observations span over two orbital periods of HD 12661 c, the current orbital period of planet c is well-constrained. An exhaustive search for libration of all possible 11:2, 6:1, and 13:2 resonant angles found that in only a few percent of all simulations performed, large amplitude libration lasts over several 10^5 yr. The current orbital elements of the two giant planets imply that the system lies near the boundary of circulating and librating regimes, regardless of the unknown orbital inclinations. This behavior causes the eccentricity of planet c to spend most of its time with a significant eccentricity ($e_c \geq$ 0.2). By computing the projected apsidal angle on the invariable plane, we find that the RMS $\Delta\varpi'$ ranges from $68° - 85°$ and MAD $\Delta\varpi'$ ranges from $63° - 82°$, for any stable initial relative inclination, and planet c's eccentricity would be greater than its current value 96% of the time.

The most straightforward interpretation is that the

eccentricity of HD 12661 b was excited by an impulsive perturbation and planet c's eccentricity is periodically excited via secular perturbations, similar to the history of the υ And c & d system (Malhotra 2002; Ford et al. 2005). Such an impulse could be the result of planet-planet scattering involving at least one additional planet (e.g. Rasio & Ford 1996; Weidenschilling & Marzari 1996). The current eccentricity of planet b suggests that the putative scattered planet would have had a mass roughly half that of planet b (Ford & Rasio 2008). If the additional planet had been scattered outwards, then there is a significant chance that it would have altered the secular evolution of planet c (Barnes & Greenberg 2006b). Given the semi-major axis of planet b, it is possible that the scattered planet was accreted onto the star. If the instability occurred after the star had reached the main sequence, then this instability could contribute to the star's large surface high metalicity. One potential concern for this mechanism is whether the timescale for the apoastron of the scattered planet to be lowered would be significantly shorter than the $\simeq 10^4-10^5$yr timescale for secular evolution of planets b & c.

There is an alternative interpretation of our results. One might think that we are unlikely to observe a system at such a rare time, casting doubt upon the orbital fit and thus the inferred secular evolution. One would expect that one of the 30 currently observed multiple planet systems is transiently passing through such an unusual state. However, there are several ways in which a system could be "unusual" so that the standard caveats of *a posteriori* statistics apply. Given the current radial velocity observations, there are few alternatives. We consider it unlikely that our global search missed a qualitatively different pair of orbits due to our assumption of non-interacting Keplerian orbits. More plausibly, one or more undetected planets could be affecting the orbital solution and/or the secular evolution of the system. After accounting for planets b & c, we find no statistically significant periodicities in the current radial velocity observations. Current observations are consistent with a small long-term acceleration, but zero acceleration falls within the 95% credible interval. Further, models with a long-term acceleration do not result in significantly different orbital parameters. Our Bayesian analysis assuming two planets and uncorrelated Gaussian noise constrains the jitter to be $\simeq 3.4 \pm 0.7$m/s. Further radial velocity observations can test whether a portion of this jitter is due to additional planets.

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TABLE 1
Results of N-body Integrations

i_{rel}	(RMS) aligned fraction	(MAD) aligned fraction	RMS $\Delta\varpi'$	MAD $\Delta\varpi'$	Stable	Isotropic
$0°$	35.9%	31.7%	$84.9° \pm 8.5°$	$81.1° \pm 7.8°$	100%	...
$0° - 30°$	39.3%	35.2%	$83.8° \pm 9.4°$	$79.5° \pm 8.6°$	99.7%	6.7%
$30° - 60°$	56.8%	51.8%	$84.2° \pm 10.4°$	$79.7° \pm 9.8°$	93.3%	18.3%
$60° - 90°$	56.0%	52.0%	$83.8° \pm 10.5°$	$79.6° \pm 9.9°$	16.4%	25.0%
$90° - 120°$	—	—	—	—	0.2%	25.0%
$120° - 150°$	95.2%	95.2%	$74.7° \pm 12.1°$	$69.3° \pm 11.1°$	16.7%	18.3%
$150° - 180°$	99.6%	99.2%	$68.4° \pm 12.8°$	$63.0° \pm 12.1°$	98.6%	6.7%
Prograde, isotropic					69.8%	80.0%
Retrograde, isotropic					38.5%	50.0%

Note. — Columns 2 and 3 each list the RMS and MAD percent of stable systems with a libration center of $0°$ instead of $180°$ for the range of relative inclinations in column 1. Columns 4 and 5 report the RMS and MAD variations for systems which librate around the most common center for that range of initial relative inclinations. Column 6 lists the percent of stable systems, and Column 7 lists the percent of systems with the given range of inclination for an isotropic distribution. The bottom two rows present aggregated stability statistics for the isotropic prograde and retrograde distributions of orbits. 2000 simulations were performed for each prograde relative inclination bin, and 500 simulations for each retrograde bin.